The new structural concept Tensairity: Basic principles

R.H. Luchsinger
Prospective Concepts AG, Glattbrugg, Switzerland

A. Pedretti, M. Perdetti & P. Steingruber
Airlight Ldt, Biasca, Switzerland

ABSTRACT: Light weight structures are a challenge for the structural engineer and an important step towards a sustainable architecture. We present the new light weight structural concept Tensairity. In Tensairity, compression and tension are physically separated. Low pressure compressed air is used for pretensioning the tension element and for stabilizing the compression element against buckling. It can be shown that no buckling problem arises. This allows to use the material both for tension and compression to its yield limit. As a result, Tensairity girders can be by factors lighter than conventional beams. The technology is ideally suited for wide span structures and for deployable applications as temporary bridges, scaffolds or large tents. Prototypes, finite element analysis as well as experimental studies have proven the concept. In this paper, the basic principles of Tensairity are presented.

1 INTRODUCTION

Light weight structures are more than light materials. The essence of engineering light weight structures is a careful design of the force flow within the structure such that minimal material is used for the specific task. Cables under tension are the most efficient way of structural use, since the cable strength is independent of the length of the cable and solely given by the material strength. However, whenever there is tension, there is compression, too. And for compression length matters. The danger of buckling demands for larger cross sections and thus for more material. As a result, columns are heavier and thicker than cables as it obvious in the case of suspension bridges.

Constructive separation of tension and compression is a major goal of good light weight engineering. The principle is fully adapted in tensegrity structures (Fuller 1975, Pugh 1976). Astonishing sculptures were built with the tensegrity principle of discontinuous compression and continuous tension, but structures solely based on floating compression are not suited for technical applications (we do not consider the compression ring of cable roofs as a tensegrity structure).

In our new developed structural concept Tensairity we have overtaken the tensegrity principle of constructive separation of tension and compression in cables and struts. Plus we have added as a third element low pressured air for stabilization.

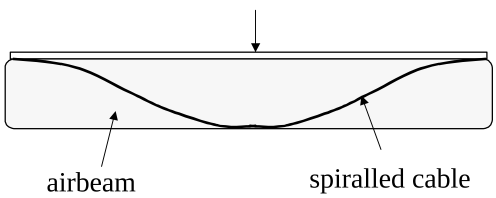

Figure 1. Basic elements of a Tensairity beam.

Cables, struts and compressed air complete each other perfectly. The result is a modified airbeam with the same load bearing capacity as a steel beam but with a dramatically reduced weight. The protected brand name Tensairity®, a combination of tension, air and integrity, reflects the relationship with tensegrity.

The main advantages of Tensairity structures are light weight, fast erection and dismantling and small storage and transportation volume. Given this properties, Tensairity is ideally suited for wide span roof structures as well as deployable structures as tents and temporary bridges.

In this paper we describe the basic elements and principles of Tensairity. A simple mathematical model is given to understand the fundamental properties of Tensairity. In a second paper presented in this volume (Pedretti et al. 2004), first applications and details about finite element modelling of Tensairity structures are outlined.

2 THE TENSAIRITY PRINCIPLE

The basic Tensairity beam consists of three major parts: a cylindrical airbeam under low pressure, a compression element tightly connected to the airbeam and two cables running with different helicity in a spiral form around the airbeam (Fig. 1). The cables are connected to each end of the compression element closing the force flow between cables and compression element. The role of the compressed air is to pretension the cables and to stabilize the compression element against buckling. In Tensairity the airbeam has solely a stabilizing function which is the reason that Tensairity can operate with low air pressure. The loads are carried by the cables and the compression element. Therefore, the load bearing capacity of a Tensairity structure is determined by the dimensions of the cables and the compression element. Since no buckling problem arises in the compression element, the material of both the cable and the compression element can be used to the yield limit and therefore in its most efficient way. This is the reason for the outstanding light weight properties of Tensairity.

3 PHYSICAL MODEL OF TENSAIRITY

3.1 *Role of the model*

The mechanics of a Tensairity beam is described by a mixture of beam theory and membrane theory. Due to the inherent three dimensional character of the structure given by the combination of spiralled cables and linear compression elements, the theory of Tensairity is complex. Therefore, numerical finite element analysis must be applied for accurate calculation of the behaviour of Tensairity in most situations (Pedretti 2004). However, to shed light on the mechanics of Tensairity, a physical model is presented in this paper. For clearness and simplicity, many simplifications and approximations underlie the model. The main objective of this model is a basic understanding of the interactions between load, pressure, membrane, compression element and cables in Tensairity beams.

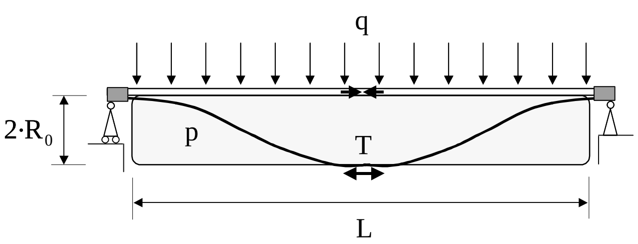

Figure 2. Simply supported Tensairity beam under homogenous distributed load.

3.2 *Cable forces*

A Tensairity beam with span L under homogenous distributed load q is shown in Figure 2. The maximal bending moment in the middle of the beam is given by standard beam theory

$$M = \frac{q \cdot L^2}{8} \quad . \tag{1}$$

The bending moment of the load is compensated by the moment given by the cable tension times the diameter of the beam. With T equal the total cable force and R_0 the radius of the beam, one finds

$$M = T \cdot 2 \cdot R_0 \, . \tag{2}$$

Introducing the slenderness g of the beam

$$g = \frac{L}{2 \cdot R_0} \quad , \tag{3}$$

the total cable force can be determined from Equation 1 and 2 to be

$$T = \frac{q \cdot L \cdot g}{8} \quad . \tag{4}$$

The total cable force is proportional to the load, the span and the slenderness of the beam.

3.3 *Membrane cable interaction*

The cable force is also determined by the interaction of the cable with the membrane. Under load, the cable presses into the membrane leading to a normal force f on the cable. From cable theory it is well known that the cable force is given by the product of the normal force and the curvature of the cable. The total cable force for two cables is then

$$T = 2 \cdot f \cdot r \tag{5}$$

with the curvature r of the spiralled cable given by

$$r = R_0 \cdot \left(1 + \frac{g^2}{p^2}\right) \cong R_0 \cdot \frac{g^2}{p^2} \quad \text{for} \quad g >> 1. \tag{6}$$

The normal force f on the cable is a function of the constriction of the membrane. The more the cable presses into the membrane, the higher the normal force. Due to the spiralled form of the cable, the interaction between cable and membrane is involved. However, with increasing slenderness of the tube, the cable lies more and more parallel to the tube axis. For parallel cables, the problem reduces to two dimensions and a simple model can be formulated. In Figure 3, the initial circular cross section of the tube (dashed line) is constricted by two diametrical forces representing the cables. As the cables press into the membrane, the initial circle of the membrane splits into two circles with decreasing radius.

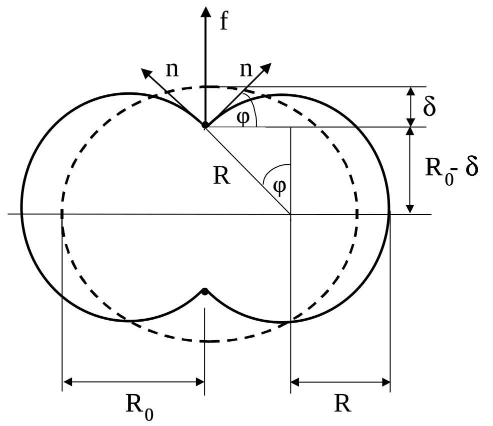

Figure 3. Membrane deflection due to the cable.

Assuming an inelastic membrane, the radius as a function of the angle j decreases as

$$R = R_0 \cdot \frac{1}{1+\frac{2 \cdot j}{p}} \quad . \tag{7}$$

The membrane hoop force n in a cylinder under pressure p is

$$n = p \cdot R \tag{8}$$

and thus the normal force on the cable

$$f = 2 \cdot n \cdot \sin j \ . \tag{9}$$

We assume that the pressure is kept constant in the tube.

By means of Equations 7, 8 and 9, the normal force can be written as

$$f = 2 \cdot p \cdot R_0 \cdot \frac{\sin j}{1+\frac{2 \cdot j}{p}} \quad . \tag{10}$$

In terms of the constriction d defined as

$$d = R_0 - R \cdot \cos j \ , \tag{11}$$

the normal force can be approximated up to first order by

$$f = p \cdot p \cdot d \ , \tag{12}$$

and up to second order by

$$f = p \cdot R_0 \cdot p \cdot \frac{d}{R_0} \cdot \left(1 - \frac{d}{R_0}\right). \tag{13}$$

The normal force (Eq. 10) and its first and second order approximation are plotted as a function of the constriction in Figure 4. As can be seen, the second order approximation is very good for $d/R_0 \leq 0.3$. Restricting the deflection to $d/R_0 \leq 0.2$, the maximal normal force is given by

$$f_{\max} \cong p \cdot R_0 \cdot 0.5 \ . \tag{14}$$

The maximal total cable force is given by (Eqs. 5, 6, 14)

$$T_{\max} = p \cdot R_0^{\ 2} \cdot \frac{g^{\ 2}}{p^{\ 2}} \quad . \tag{15}$$

Defining the load per area q_a by

$$q_a = \frac{q}{2 \cdot R_0} \ , \tag{16}$$

the pressure as a function of the load per area can be obtained from Equation 4 and 15

$$p = \frac{p^{\ 2}}{2} \cdot q_a \ . \tag{17}$$

The important result is that the pressure in the Tensairity beam is independent of the length of the beam and the slenderness. It is solely given by the load per area. For example a pressure of $p = 5$ kN/m^2 (50 mbar) results for a load per area of $q_a = 1$ kN/m^2 (100 kg/m^2).

The maximal membrane force is given by Equation 8 for $R = R_0$. Together with Equation 17 one obtains

$$n = \frac{p^{\ 2}}{4} \cdot q \ . \tag{18}$$

Again, the membrane force is independent of the length and the slenderness of the Tensairity beam and solely determined by the load per length. With a modest membrane force of $n = 25$ kN/m a load of $q = 10$ kN/m $\cong 1000$ kg/m can be carried. These investigations show, that membrane forces are well within the tolerances of standard fabrics for typical load cases as given by the construction industry. Since the strength of the cables and the compression element can be easily designed to the situation at hand, Tensairity structures can withstand any reasonable load.

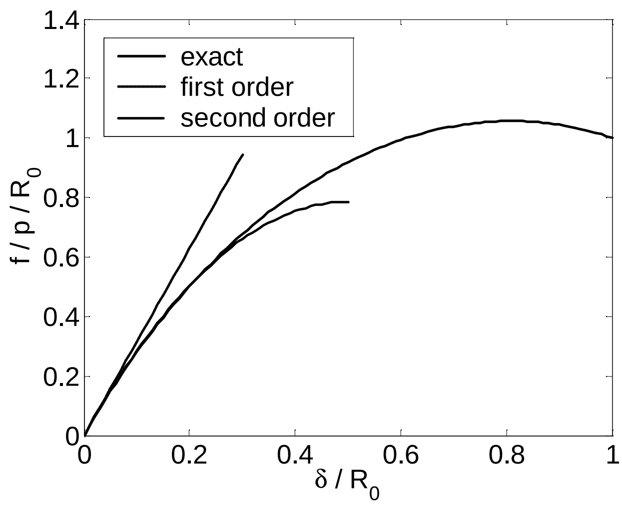

Figure 4. Scaled cable normal force as a function of the membrane constriction.

3.4 *Tensairity beam deflection*

As in standard beam theory, the Tensairity beam is deflected under load. The displacement is the sum of three factors: the lengthening of the cables due to the load stress, the length reduction of the compression element due the load and the reduction of the spiral radius of the cable due to the constriction of cable into membrane under load. Assuming that the beam bends into a circular shape, one can obtain for large g the following relation for the deflection w per length at the middle of the beam

$$\frac{w}{L} = \frac{1}{4} \cdot g \cdot \left(e_t + e_c + \frac{d}{R_0} \cdot \frac{1}{1+\frac{g^2}{p^2}} \right) \tag{19}$$

where e_t is the strain of the tensioned cable, e_c is the strain of the compression element and d the constriction of the cable into the membrane as defined by Equation 11. The maximal constriction $d/R_0 = 0.2$ used to define Equation 14 might be too large for some deflection critical applications. A higher pressure can thus be applied to reduce the beam deflection.

4 BUCKLING FREE COMPRESSION

The cable force acts on the compression element and buckling of the compression element must be considered. However, the compression element is tightly connected to the membrane and thus can be considered as a beam on an elastic foundation. The buckling load for such a beam is given by (Szabo 1977)

$$P = 2 \cdot \sqrt{k \cdot E \cdot I} \tag{20}$$

with the spring constant k of the elastic foundation (in N/m^2), the modulus of elasticity E and the moment of inertia I of the compression element. In Tensairity the spring constant can be obtained from the investigations of the interaction between cable and membrane. The compression element lies parallel to the tube axis and the two dimensional model of Figure 3 can be applied. By means of Equation 12, one obtains

$$k = \left.\frac{\partial f}{\partial d}\right|_{d=0} = p \cdot p \tag{21}$$

and the critical buckling load for the Tensairity compression element is

$$P = 2 \cdot \sqrt{p \cdot p \cdot E \cdot I} \ . \tag{22}$$

The buckling load increases with the square root of the pressure and is independent of the length of the compression element. A proper choice of the moment of inertia of the compression elements allows to push the buckling load beyond the yield load and thus making the yield load as the limiting factor of the compression element. This is what we call buckling free compression.

5 WEIGHT OF TENSAIRITY BEAMS

The mass of a Tensairity beam is given by the mass of the cables, the mass of the compression element and the mass of the membrane. The cross sectional area of the cable A_t is determined by the tension force T and the cable yield stress s_t. The mass per length is given by the product of cross sectional area with the cable mass density r_t.

The load of the compression element is equal to the tension force of the cables. Thus the material parameters s_c and r_c of the compression element define its mass per length in the same way as for the cable. The mass of the membrane is commonly defined by the mass per area w (in kg/m^2). With Equation 4 the total mass per length of a Tensairity beam is

$$\frac{m}{L} = \frac{q \cdot L \cdot g}{8} \cdot \frac{r_t}{s_t} + \frac{q \cdot L \cdot g}{8} \cdot \frac{r_c}{s_c} + w \cdot 2 \cdot p \cdot R_0 , \tag{24}$$

where the end caps of fabric tube are neglected for simplicity.

6 COMPARISON WITH SIMPLE AIRBEAMS

A simple airbeam is a fabric tube with radius R_0 under pressure without any struts and cables. Under load, the tube starts to bend and wrinkles appear when the bending moment is greater than (Comer & Levy 1963, Main et al. 1994)

$$M = \frac{1}{2} \cdot p \cdot p \cdot R_0^3 . \tag{25}$$

Although the collapse moment is twice the wrinkling moment, deflections increase considerably above the wrinkling moment . Thus, the wrinkling moment can be considered as a reasonable load limit for practical applications. Together with Equation 1 and 16, the pressure in the tube p to hold a given load per area q_a is given by

$$p = \frac{2}{p} \cdot q_a \cdot g^2 \ . \tag{26}$$

The ratio of the load bearing capacity of a Tensairity beam at given pressure (Eq. 17) to a simple airbeam at the same pressure is then

$$\frac{q_{a,Tensairity}}{q_{a,airtube}} = \frac{4}{p^3} \cdot g^2 . \tag{27}$$

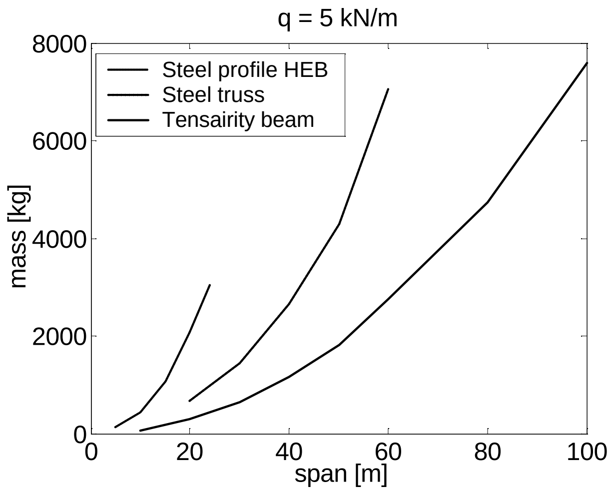

Figure 5. The mass of HEB steel profile girders, steel trusses and Tensairity beams as a function of span for a given load case.

For $g = 10$ one obtains $q_{a,Tensairity} / q_{a,airtube}$ = 13, for $g = 30$ one obtains $q_{a,Tensairity} / q_{a,airtube}$ = 116. Depending on g, the Tensairity beam is ten to one hundred times stronger than a simple airbeam with the same dimensions and pressure. Or to put it differently, the pressure in a simple airbeam must be ten to one hundred times larger then in a Tensairity beam to withstand the same load. Therefore, applications of simple airbeams in construction is very limited.

7 EXAMPLES

As an example, we consider a Tensairity beam with span $L = 10$ m, slenderness $g = 10$ and a load per area of $q_a = 1$ kN/m^2. The load per length is $q = 2$ kN/m (Eq. 16). The needed pressure is 50 mbar (Eq. 17) and the membrane force is 4.9 kN/m (Eq. 18). The mass of the beam is given by Equation 24. For steel cables ($s_t = 15 \cdot 10^8$ N/m^2, $r_t = 7800$ kg/m^3), a steel compression element ($s_c = 3.55 \cdot 10^8$ N/m^2, $r_c = 7800$ kg/m^3) and a membrane with mass per area of w = 0.8 kg/m^2 (PVC/Polyester type I), one obtains 0.13 kg/m for the cables, 0.55 kg/m for the compression element and 2.5 kg/m for the membrane. The whole beam weights 32 kg. The membrane with 25 kg is the most important part. A steel profile (HEB 100) with the same span designed for the same load weighs 204 kg. Tensairity is by a factor 6 lighter than the steel girder.

Increasing the load by the factor 10 to $q = 20$ kN/m, the pressure increases to 500 mbar and the membrane force to 49 kN/m. With the same materials for the cables and the compression element, however with a stronger membrane (PVC/Polyester type IV, w = 1.3 kg/m^2) one gets masses of 1.3 kg/m (cables), 5.5 kg/m (compression element) and 4.1 kg/m (membrane). The total mass of the 10 m span Tensairity beam is 109 kg. Now the heaviest contribution results from the compression element. The steel profile (HEB 220) weights 715 kg and a similar weight ratio to Tensairity as for the load case $q = 2$ kN/m results.

A more detailed comparison of the weight of steel profiles (HEB), steel trusses and Tensairity beams as a function of the span is shown in Figure 5. A distributed load of 5 kN/m was applied, the slenderness of the Tensairity beam is 20. As can be seen, Tensairity is in the given range between a factor two and three lighter than the truss, which itself is about a factor three lighter than the steel profile.

8 CONCLUSIONS

The fundamental mechanics of the Tensairity beam can be understood by a simple physical model connecting beam mechanics and membrane physics. The crucial property of Tensairity is that important parameters as the pressure and the membrane force are solely given by the load and independent of the length and slenderness of the beam. Due to the elastic embedding of the compression element, buckling free compression can be realized. Thus, the material is used in the most efficient way both for tension and compression and the Tensairity structure is by factors lighter than conventional beams. The Tensairity technology has the advantages of easy storage, easy transportation and easy erection. Indeed, about 95 % of the Tensairity beam volume is air, which does not need to be transported. Sharing the advantages with the simple airbeam, the load bearing capacity of Tensairity is by orders of magnitudes higher than for the simple airbeam. When compared to conventional steel structures, Tensairity beams can withstand the same loads by a dramatically reduced weight. Naturally, more sophisticated materials as carbon or Kevlar can be used in Tensairity when demanded. With all these extraordinary properties, Tensairity has a tremendous potential in construction industry for applications as e.g. wide span roof structures, temporary bridges and temporary buildings.

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